USN

10CS42

Fourth Semester B.E. Degree Examination, Dec.2016/Jan.2017
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define a graph and degree of a vertex of a graph. Prove that in every graph the number of vertices of odd degree is even. **(06 Marks)**

b. Define self-complementary graph. How many edges must G have, if G is a self-complementary graph? Give one example for each of the self complementary graph on 4 vertices and 5 vertices. **(07 Marks)**

c. Show that a connected graph with exactly 2 vertices of odd degree has an Euler Trail. Find the Euler circuit in the graph shown below. **(07 Marks)**

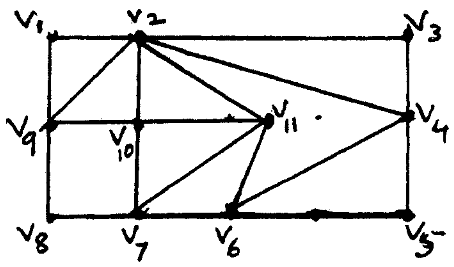

Fig.Q.1(c)

2 a. State Euler's fundamental theorem on planar graphs. Verify the same for the following graph. Also construct the dual of the same graph. Fig.Q.2(a). **(06 Marks)**

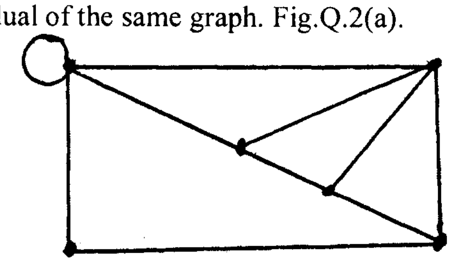

Fig.Q.2(a)

b. Check the planarity of the following graph by the method of elementary reduction Fig.Q.2(b). **(07 Marks)**

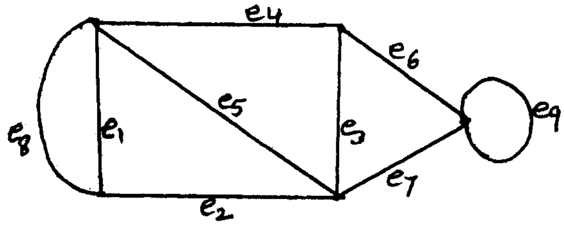

Fig.Q.2(b)

c. Define chromatic number and chromatic polynomial of a graph. Find the chromatic polynomial for the cycle C_4. What is its chromatic number? **(07 Marks)**

3 a. Define a tree and a forest. Prove that a tree with two or more vertices contains at least 2 leaves. Further, show that if a tree has exactly two pendent vertices, the degree of every non-pendant vertex is two. **(06 Marks)**

b. Show that a Hamilton path is a spanning tree. Draw all the spanning trees of the graph Fig.Q.3(b). **(06 Marks)**

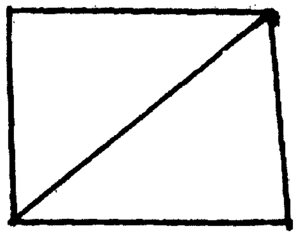

Fig.Q.3(b)

c. Construct an optimal prefix code for the letters of the word 'ENGINEERING'. Hence deduce the code for this word. **(08 Marks)**

4 a. Apply Prim's algorithm to find a minimal spanning tree for the graph Fig.Q.4(a). **(07 Marks)**

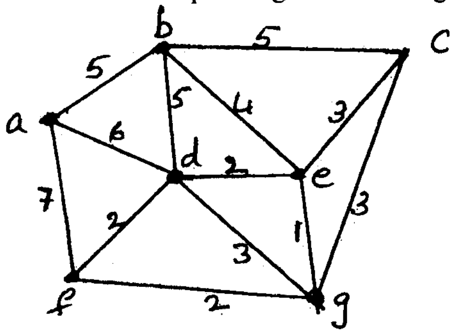

Fig.Q.4(a)

b. Apply Dijkstra's algorithm to the weighted digraph, to find the shortest distance from vertex 1 to each of the other vertices Fig.Q.4(b). **(08 Marks)**

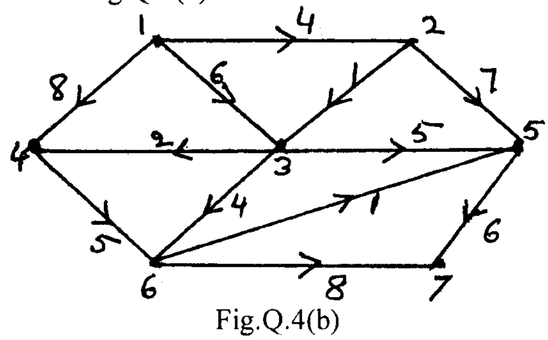

Fig.Q.4(b)

c. Define matching. Five students s_1, s_2, s_3, s_4, s_5 are members of 3 committees c_1, c_2, c_3. The committee c_1 has s_4 and s_3 as members, the committee c_2 has s_1, s_3, s_5 as members and the committee c_3 has s_2 and s_5 as members. Each committee is to select a student representative. Can a selection be made in such a way that each committee has a distinct representative? **(05 Marks)**

PART – B

5 a. How many arrangements are there for all the letters in the word 'SOCIOLOGICAL'? In how many of these arrangements i) A and G are adjacent? ii) All the vowels are adjacent? **(07 Marks)**

b. Find the coefficient of i) x^{12} in the expansion of $x^3(1-2x)^{10}$ and ii) $x^2y^2z^3$ in the expansion of $(3x-2y-4z)^7$. **(06 Marks)**

c. Define Catalan number. Using the moves R: $(x, y) \to (x+1, y)$ and u: $(x, y) \to (x, y+1)$ find in how many ways can one go,

i) From (0, 0) to (6, 6) and not rise above the line $y = x$?

ii) From (2, 1) to (7, 6) and not rise above the line $y = x - 1$? **(07 Marks)**

6 a. Find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$ under the condition $x_i \le 7$ for $i = 1, 2, 3, 4$. **(07 Marks)**

b. There are n pairs of children's gloves in a box. Each pair is of a different colour. Suppose the right gloves are distributed at random to n children and there after the left gloves are also distributed to them at random. Find the probability that,

i) No child gets a matching pair.

ii) Every child gets a matching pair.

iii) Exactly one child gets a matching pair. **(06 Marks)**

c. Find the rook polynomial for the board shown below (shaded part). **(07 Marks)**

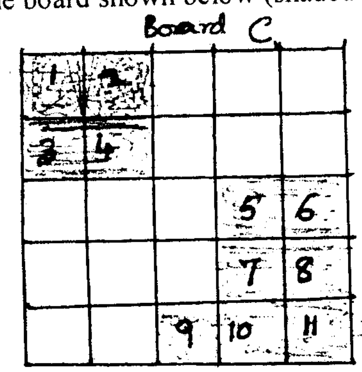

Fig.Q.6(c).

7 a. Using generating function, derive the formula $\sum_{k=0}^{n} k^3 = \left\{\frac{n(n+1)}{2}\right\}^2$. **(07 Marks)**

b. In how many ways can 12 oranges be distributed among 3 children A, B, C so that A gets at least 4, B and C gets at least 2, but C gets no more than 5? **(07 Marks)**

c. A company appoints 11 software engineers, each of whom is to be assigned to one of four offices of the company. Each office should get at least one of these engineers. In how many ways can these assignments be made? **(06 Marks)**

8 a. Find the recurrence relation and the initial condition for the sequence 0, 2, 6, 12, 20, 30, 42, … Hence find the general term of the sequence. **(06 Marks)**

b. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \ge 2$ given that $a_0 = -1$ and $a_1 = 8$. **(07 Marks)**

c. Find the generating function for the recurrence relation, $a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \ge 0$ and $a_0 = 3$, $a_1 = 7$. Hence solve it. **(07 Marks)**

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